A METRIC-BASED APPROACH TO 2D TOOL-PATH OPTIMIZATION FOR HIGH-SPEED MACHINING

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ABSTRACT

Conventional tool-path generation strategies are readily available to generate geometrically feasible trajectories. Such approaches seldom take into consideration physical process concerns or dynamic system limitations. In the present work, an approach for improving a geometrically feasible tool-path trajectory based on quantifiable process metrics is developed. Two specific measures of toolpath quality are incorporated into the iterative improvement algorithm: instantaneous path curvature and instantaneous cutter engagement. These metrics are motivated by a desire to minimize acceleration requirements and maintain a stable steady-state cutting process during high-speed machining. The algorithm has been implemented for two-dimensional end-milling operations, and case studies are presented to illustrate the approach.

INTRODUCTION

High speed machining (HSM) is a key enabling technology in an increasing number of industries. In the aerospace industry, structural components are increasingly being machined as monolithic structures from a single billet. The result is drastically reduced part counts, assembly costs, and even maintenance costs. The Boeing F/A 18 E/F tactical aircraft realized a 42% reduction in parts and a 25% weight savings over previous models, attributed not in small part to the design changes made practical by the application of high speed machining technology [Zelinski, 1999]. In the tooling industry, high speed machining technology continues to grow increasingly important for maintaining economic competitiveness. Successful applications of HSM to the production of tooling for forging, extrusion, sheet forming, die casting, and injection molding have been reported [Fallböhmer et. al., 2000; Dewes and Aspinwall, 1997; Gough, 1990; Allcock, 1994].

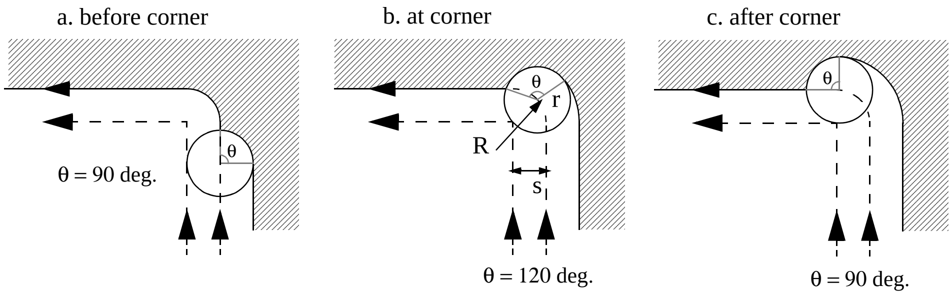
High-velocity (HV) machine tools with 1,000+ ipm traverse rates, and 50,000+ RPM high-power spindles, are readily available. However, the velocity capabilities of such machines are seldom reached in industrial practice. Often, conservative feedrates are employed in contouring due to concerns arising from process stability and acceleration limitations. To address these issues, significant attention has been focused on the important problems of dynamic modelling, parameter optimization, and feedrate scheduling. Often overlooked, however, is the potential for improvement in process efficiency through changes to the tool-path itself.

Tool-path planning has been traditionally approached from a purely geometric perspective. The vast majority of bulk material removal occurs in 2.5D roughing operations. For such operations, tool-path trajectories are generated through conventional strategies such as contour-parallel or direction-parallel offsetting. When the dynamics and mechanics of the process and machine tool system are considered, conventional tool-path generation techniques are observed to be far from ideal. In the present work, an approach is developed to optimize a geometrically feasible toolpath based on quantifiable metrics driven by important technological concerns.

Technical challenges in HSM

Numerous technological issues in high speed machining (HSM) pose particular challenges from the perspective of process planning, optimization, and control. Conventional tool-path strategies fail to adequately address concerns such as dynamic stability, acceleration demands, and constantly varying tool engagement that limit production rates, increase tool wear, and reduce part quality.

First, and possibly foremost, are limitations imposed by consideration of dynamic stability. Dynamic stability issues drive the majority of efforts to effectively implement high speed machining on the shop floor. Tlusty states: “It has long been recognized that the occurrence of chatter is one of the most significant limitations in increasing the metal removal rate in milling” [Tlusty, et. al., 1996]. The difference between successful stable cutting and catastrophic unstable cutting can be traced to subtle interactions between the dynamics of the tool, spindle, and workpiece, instantaneous chip loads and cutter engagements. Numerous models have been developed for dynamic



$$\theta_{max} = \pi - \mathrm{acos}\left(\frac{2r^2 + 2R(s-r) - s^2}{2r(R-r)}\right)$$

Figure 1. Varying engagement arising with contour-parallel offsets

simulation of the machining process, and serve as increasingly accurate tools in selection of values for the spindle speed and cutting parameters for chatter-free machining [Altintas and Budak, 1995; Tlusty et. al., 1996; Davies et. al. 1998; Zhao and Balachandran, 2001]. Additionally, systems have been developed for avoidance or detection and control of chatter by varying the (constant) spindle speed or cutting with a varying spindle speed [Smith and Tlusty, 1992; Winfough and Smith, 1995; Jayaram et.al., 2000]

A second crucial issue in high speed machining is making effective use of the dynamic capabilities of the machine tool, actuators, and servo drives. Renton and Elbestawi [2000] demonstrate that by making complete use of the performance envelope of each servo controlled axis, feed rates can be increased while simultaneously decreasing tracking path error. Richard Bertsche, of Bertsche Engineering, a manufacturer of high velocity machine tools, emphasizes the crucial role that acceleration capabilities play in the high speed machining process [Hogan, 1999]. He notes that two machines with similar velocity yet different acceleration capabilities may differ by 50% in total processing time when cutting a moderately sized pocket. For many practical machining tasks, a high-velocity machine tool may spend the majority of its time accelerating and decelerating [Yan et. al., 2000]. During the inevitable periods of dwelling, tool temperatures climb sharply, drastically reducing tool life. Fatigue and wear of the machine tool itself is unnecessarily hastened by the extreme acceleration demands placed upon the actuators and structure. In such cases, a path that minimizes acceleration demands can have tremendous impact on cycle time.

Instantaneous Parameter Optimization

In 2.5D milling, the angle of engagement between the tool and workpiece, θ, is a convenient substitute for the radial depth of cut, as is it is unambiguous for arbitrary tool trajectories (Figure 1). The angle of engagement, axial depth of cut, cutting (spindle) speed, and instantaneous feed-rate (chip-load) uniquely determine the chip geometry, and entry/exit angles [Stori and Wright, 2000]. In conjunction with cutting tool, workpiece, and spindle/machine tool characteristics, it is these instantaneous process parameters that will determine cutting forces, deflections, and process stability.

Numerous researchers have approached the problem of optimization / selection of these instantaneous process parameters for traditional and high-speed machining. Tlusty et. al. [1990] and Smith, et. al. [1991] address issues of stability in end milling and the importance of selecting the proper axial and radial depths of cut for chatter-free machining. Stori et. al. [1999, 2001] developed a simulation-based approach for parameter selection based on static model predictions of cutting forces, power, and tool deflection. A review of prior approaches to parameter optimization may be found in Stori et. al. [1999].

Feedrate Scheduling and Toolpath Modification

Despite the sensitivity of these instantaneous parameters to process stability and efficiency, there has been surprisingly little work done to modify tool-paths explicitly, such that favorable conditions of these instantaneous parameters may be achieved in the cutting of complex geometries. As the axial depth of cut and cutting speed are not readily varied within a continuous cutting operation with a flat end-mill, the engagement angle and instantaneous feed-rate become critical process control variables. The feed-rate may be scheduled independently of tool-path geometry, and a number of researchers have explored feed-rate optimization / scheduling as a post processing step after a trajectory has been generated. Recently, Renton and Elbestawi [2000], Farouki et.al. [2000], and Stori and Ferreira [2002] have proposed feed-rate scheduling approaches driven by axis velocity and acceleration limitations. Additionally, methods have been developed to schedule feedrates based on model predictions of constraints such as forces and tool deflections [Wang, 1988; Yang and Sim, 1993; Fussell et.al., 2001].

However, the potential for improvement in engagement variation and acceleration demands through changes to the tool-path geometry is significant. In only a very small number of instances has reference been made to explicit attempts to modify the toolpath geometry based on physical process concerns. Tsai et. al. [1991], add additional circular arc segments to convex inside corners to keep cutter engagement below prescribed limits. They attribute this technique to Iwabe et. al [1989]. Tlusty et. al. [1990] and Smith, et. al. [1991] avoid over-engagement in cornering by adopting a spiral-in approach.

METRICS FOR TOOL-PATH EVALUATION

One recurring challenge in process planning activities is the difficulty of concretely evaluating the quality and efficiency of competing alternative solutions. Planning decisions are often ad hoc, and approaches to CAPP typically rely on heuristics and rule structures for decision making. The notion of optimality is generally avoided, perhaps due to the extreme breadth of the solution space.

In the present work, an approach based on *quantifiable metrics* for tool-path evaluation is developed. While it may yet remain not possible to claim optimality of tool-path trajectories, the introduction of an approach based on quantifiable metrics will permit unbiased evaluation of algorithmic progress and comparison with conventional approaches.

The developed metrics for tool-path evaluation are based on the following assumptions:

- The ideal cutting operation is a steady-state process, and a target operating point for the process can be identified a priori. This instantaneous cutting state target is characterized by an instantaneous engagement angle, feed, cutting speed, and depth of cut.
- It is desired to maintain the cutting process at, or near, this ideal operating point for as much of the operation as possible. The larger the deviation from the ideal operating condition, the more inferior the solution, and the longer the deviation from the ideal operating point, the more inferior the solution.

Given the above criteria, a path integral consisting of individual terms penalizing deviations from the ideal operating point for each of the instantaneous state variables represents a natural functional form for the metric. This approach has an analogy with the so-called weighted-norm methods often used to scalarize a multiple objective optimization problem [see Li et. al. 1999 for a review of such methods]. In our case, however, the penalty is integrated along the entire path. If the path is represented as a sequence of j continuous segments parameterized by a variable u, and $\overline{z_{ij}}$ represents target values of each of the i state variables (instantaneous engagement, feed, etc.), then the penalty function takes the form of Equation 1, below. The scalars w_i are the weights, and the exponent p can vary from 1 to ∞. Typically, a value of p=1 or 2 would be commonly used. Note that as p increases, the solution approaches the weighted minimax, or Tchebycheff method (minimize the maximum deviation of any single variable from the target).

$$U = \sum_i \sum_j \int_u (w_i |f_i(u_j) - \overline{z_{ij}}|^p du) \quad (1)$$

In the above expressions, $f_i(u_j)$ is a function that returns the instantaneous state at the point u_j, $\overline{z_{ij}}$ is the target state value, j is the segment number, and i is the specific technical metric considered. Within the general framework described in Equation 1, a variety of efficiency, quality, and process metrics can be supported.

Curvature. Path curvature is integrally tied to the acceleration and jerk required to track the required trajectory. In the high speed machining domain, it is desirable to minimize the actuator requirements, and time spent accelerating. In the extreme case of discontinuous cornering, the inherent dwelling leads to rapid unloading and loading of the machine axes and cutting tool, resulting in decreased part quality and excessive tool and machine wear. Typically, the individual axes will have independent dynamic characteristics. We assume the path geometry is represented by a combination of individual continuous segments parametrized by a variable, u, $\mathbf{P}(u) = (x(u), y(u), z(u))$. The path geometry must be augmented with a mapping into the time domain in order to achieve an intended feed rate profile. This mapping may be represented as an additional function $u(t)$, prescribing the parametric speed.

The acceleration vector is obtained through application of the chain rule, and differentiation:

$$\mathbf{a} = \frac{d^2}{dt^2}\mathbf{P}(u(t)) = \mathbf{P}''(u(t))\dot{u}(t)^2 + \mathbf{P}'(u(t))\ddot{u}(t) \quad (2)$$

where the dot operator represents differentiation with respect to time, and the prime operator denotes differentiation with respect to the parameter u. In order to make the relationship between feedrate and geometric curvature explicit, the acceleration of a single axis (x) may be expressed in the following form [Chou and Yang, 1991]:

$$a_x = V^2\kappa_x + \dot{V}t_x \quad (3)$$

where $V = \|\mathbf{P}'(u)\dot{u}\|$ is the tangential velocity (feedrate), κ_x is the x component of the curvature of the path, $\dot{V}$ is the feed-acceleration, and t_x is the x component of the unit tangent vector, $\mathbf{t} = \mathbf{P}'(u)/\|\mathbf{P}'(u)\|$. The curvature, κ, can be expressed as [Chou and Yang, 1991]:

$$\kappa = \kappa_x(u)\mathbf{i} + \kappa_y(u)\mathbf{j} + \kappa_z(u)\mathbf{k} = \frac{\frac{d\mathbf{P}(u)}{du} \times \frac{d^2\mathbf{P}(u)}{du^2} \times \frac{d\mathbf{P}(u)}{du}}{\left|\frac{d\mathbf{P}(u)}{du}\right|^4} \quad (4)$$

Note that the curvature is solely a function of the path geometry, $\mathbf{P}(u)$, and can be represented analytically. In this case, the desire would be to minimize the curvature and hence acceleration demands, $f(\underline{u_{i1}}) = \|\kappa\|$ Correspondingly, the target curvature would be 0, or $\overline{z_{i1}}(u) = 0$

Instantaneous Cutter Engagement. Varying cutter engagement is an inevitable consequence of parallel offsets, as has been observed by numerous researchers (Figure 1).The instantaneous cutter engagement, θ, is a function of the continuously changing in-process geometry of the workpiece. The engagement angle is therefore dependent on the *history* of the cutter path, and evaluation of the instantaneous cutter engagement must be carried out via geometric process simulation. Assuming that a target instantaneous cutter state can be identified at a given point in time for a particular tool-path trajectory, it is desired to minimize the deviation from this target, that is:

$$f_{i2}(u) = |\theta_i| \qquad \overline{z_{i2}}(u) = |\theta_t| \quad (5)$$

where θ_t is the target engagement angle and θ_i is the engagement angle obtained via simulation. The selection of an optimal instantaneous engagement angle should be based on dynamic stability, tool wear, and tool deflection concerns.

Multiple-Objective Planning Metric. Within the general framework of Equation 1, several disparate metrics may be combined with appropriate weights to form a single utility, or cost function. For example, weights w_1 and w_2 may be assigned to the curvature, and engagement angle metrics respectively according to application requirements, and the following cost function is obtained:

$$U = \sum_i^2 \sum_j^n \int_0^1 w_i |f_{ij}(u) - \bar{z}_{ij}(u)| du \quad (6)$$

where n is the number of continuous segments comprising the tool-path.

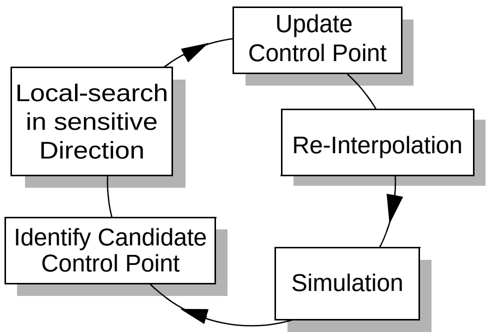


Figure 2. Iterative Improvement Algorithm

Additional Planning Metrics. The above are not intended to constitute a comprehensive set of planning metrics for high speed machining. Rather, the they serve to illustrate the types of concerns that may be considered with a metric-based approach. Below, we mention several other metrics that might find relevance for particular applications:

- rewarding increases in the distance from the process stability frontier, intended to provide robustness to the plan in the face of varying system dynamics.
- penalizing entry and exit angles most susceptible to tool damage and breakage.
- rewarding certain variations in uncut chip thickness, a technique useful for prolonging tool life.
- penalizing small chip thicknesses and low feed velocities that lead to excessive temperature increases.

METRIC-BASED IMPROVEMENT APPROACH

The majority of volumetric material removal in milling is achieved through 2D roughing operations with a flat end-mill. For such operations, the periphery geometry is often rigidly constrained. Geometrically feasible toolpaths may be readily generated for such operations through one of two broad techniques: the contour-parallel (spiral in/out) or direction-parallel (zig-zag) approach. We adopt the topology of the contour-parallel tool-path as a starting point for the algorithm. The intent is to optimize the trajectory of the geometrically unconstrained (interior) tool-path segments so as to minimize deviations from the target instantaneous engagement, and reduce the total curvature.

The methodology developed for metric-based improvement of a geometrically feasible tool-path is outlined schematically in Figure 2. The toolpath is defined by a relatively compact set of interpolatory control points. By moving individual control points, it is possible to make changes to the local toolpath geometry. The algorithm proceeds by sequentially moving individual control points to reduce the metric function cost.

A single iteration of the algorithm consists of updating each of the control points exactly once. During each updating procedure, a local search is performed to determine the movement distance that minimizes the cost function. The control points are updated in a sequence corresponding to their individual contribution to the total path-integral cost. Below, we detail the tool-path representation, engagement simulation methodology, and the individual steps of the iterative improvement algorithm.

Tool-path Representation

An interpolatory, nearly arc-length parameterized quintic-spline (NAPQS) developed by Wang and Yang [1993] has been used to represent the tool-path during optimization. This particular formulation provides several attractive properties:

- Any continuous feasible path geometry may be approximated to an arbitrary degree o
- ints. Discontinuous paths may be accommodated through a combination of neighboring quintic spline segments.
- Changes to the path geometry through movement of interpolation points are intuitive. Movement of a single interpolation point results in a modification to the tool-path that is primarily localized to a neighborhood of five interpolation points.
- The NAPQS has second order, C^2, continuity. As a result, the corresponding single-axis acceleration commands will be continuous, an important consideration for high-speed trajectory tracking. The tangent and curvature of the spline may be analytically obtained for use in the iterative algorithm.
- The quintic spline shares a shape very similar to that of the traditional chordal-length parameterized cubic spline (CPCS). However, the two extra degrees of freedom in the NAPQS are utilized to much better preserve unit-tangency of the spline. This is a convenient property due to the extensive simulation employed in the current approach. Specifically, the equally spaced points required for accurate simulation of the instantaneous cutter engagement may be readily generated.

While the first three properties are shared by both the CPCS and NAPQS approaches, the benefits provided by near-arc length parameterization motivated the use of the NAPQS for the current application. Below, we briefly highlight some details of the interpolation procedure. For complete details, the reader is directed to Wang and Yang [1993].

A segment of a quintic spline can be defined by the following equation:

$$\mathbf{P}_i(u) = \mathbf{a}_i u^5 + \mathbf{b}_i u^4 + \mathbf{c}_i u^3 + \mathbf{d}_i u^2 + \mathbf{e}_i u + \mathbf{f}_i \quad u \in [0, L_i] \qquad (7)$$

where u is the position parameter, L_i is the range of u, $\mathbf{a}_i, \mathbf{b}_i, \mathbf{c}_i, \mathbf{d}_i, \mathbf{e}_i, \mathbf{f}_i$ are the coefficients of the spline. To uniquely define an individual spline segment, the parameter range must be specified as well as a number of geometric constraints. Typically, a spline segment is defined by constraints at its boundaries. The NAPQS interpolation approach makes use of the start and end points as well as the first and second derivatives evaluated at the beginning and end of a segment, $\mathbf{P}'_i(u)|_0$, $\mathbf{P}'_i(u)|_{L_i}$, $\mathbf{P}''_i(u)|_0$, and $\mathbf{P}''_i(u)|_{L_i}$.

To obtain the first and second derivatives at the segment boundaries, a C^2 continuous cubic spline with arc-length parameterization is generated to interpolate the given points. Renner's method [Renner, 1991] is used to approximate the initial and final

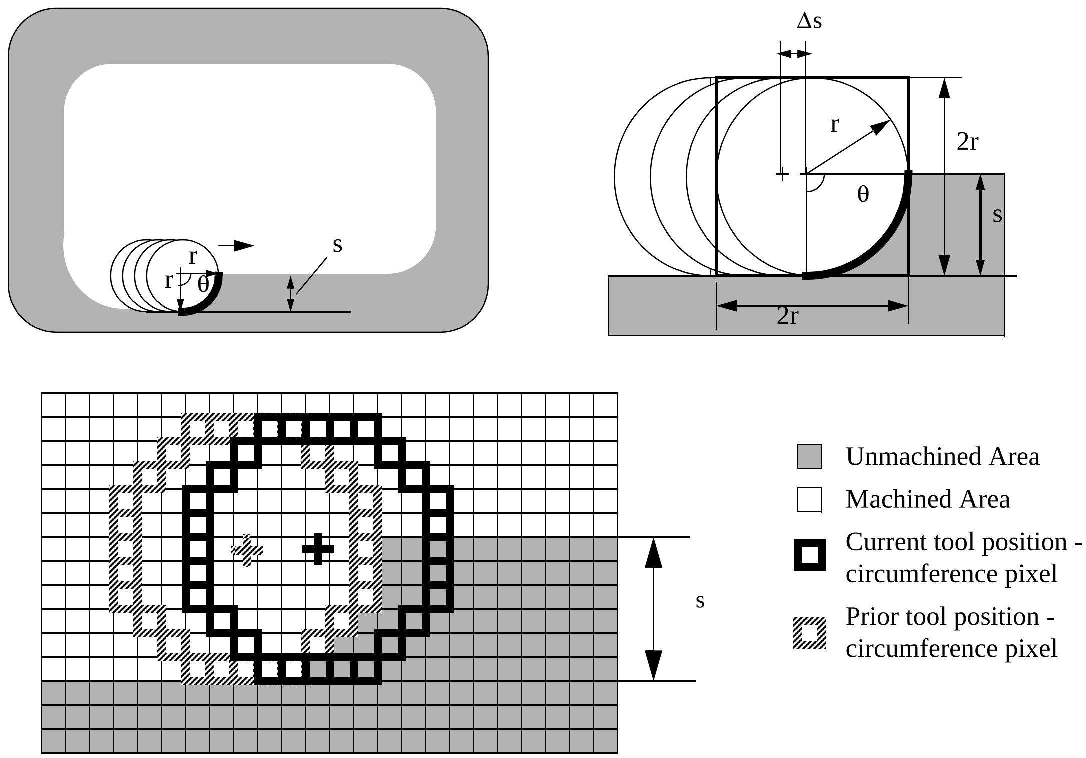


Figure 3. Pixel-based Engagement Simulation

tangent vectors. The tangents are then normalized, imposing unit-tangency at the segment boundaries. Chordal length parameterization typically results in significant deviations from unit-tangency. To approximate the ideal arc-length parameterization, an additional constraint is introduced:

$$\left|\frac{d}{du}\mathbf{P}\left(\frac{L_i}{2}\right)\right| = 1 \tag{8}$$

The addition of unit-tangency constraints at the segment midpoints and boundaries uniquely constrains the quintic spline and defines the interpolation procedure.

Engagement Simulation

Geometric simulation is required to estimate the instantaneous cutter engagement. It is important to note that cutter engagement cannot, for arbitrary tool-path trajectories, be computed analytically. This is because the cutter engagement at a particular point in time is critically dependant on the in-process geometry, which is a function of the *history* of the tool-path. Figure 3 illustrates the pixel-based simulation procedure for cutter engagement. In the figure, the gray region represents material remaining to be machined. The interior (white), region has already been machined, and the present cutter engagement, θ, is to be estimated. As illustrated in the figure, both the local in-process geometry and tool are discritized. The in-process geometry has been generated by mapping the pixels for all previous tool positions to the machined (white) state. The instantaneous engagement can be estimated by checking the status of the in-process geometry for the pixels that overlap the circumference of the cutting tool at its present location.

Fortunately, the above process can be implemented efficiently in terms of both memory utilization and computational time. There are two primary steps in the simulation process; masking the individual prior tool positions onto the in-process geometry, and reading the state of the circumference pixels for the current tool position.

It is necessary to generate a mask for the tool geometry only once. This bitmap contains the filled tool geometry (circle). A second bitmap must be initialized to the initial workpiece geometry (trivially zero if a pocket is being cut into a new surface). Masking the tool bitmap onto the workpiece bitmap requires translating the tool to the proper location, and replacing the corresponding elements of the workpiece with the result of a logical “OR” operation between the tool and workpiece. Reading the state of the engagement may be efficiently implemented with a preprocessed list of circumference pixels. These pixels are read, and the engagement tallied. Both of these tasks require only very low-level memory management and computational tasks. For these particular computational operations, special-purpose graphics hardware is not necessary.

Curvature Computation

In contrast to the computationally expensive simulation required for instantaneous engagement estimation, the path curvature may be computed directly from the quintic spline parameters. For a two-dimensional spline trajectory confined to the x-y plane, Equation 4 may be simplified as follows:

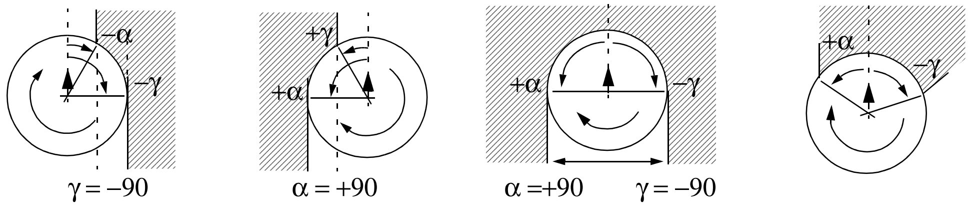


Figure 5. Entry (α), and Exit (γ) Angles in End Milling

$$\kappa = \frac{|\mathbf{P}'(u) \times \mathbf{P}''(u)|}{\|\mathbf{P}'(u)\|^3} \tag{9}$$

For the purpose of evaluating the metric cost, the above expression is evaluated at each of the discrete simulation points, and the resulting summation approximates the path integral of Equation 6. The absolute value of the signed curvature is used when computing the cost.

Sensitivity Analysis and Updating of Control Points

A control point is selected for updating based on a rank sorting of the costs obtained through simulation. The control point associated with the greatest cost is flagged for updating. The control point is moved a direction perpendicular to its tangent in the direction of descent relative to the cost function, as illustrated in Figure 4. The distance moved is a scalar u, and the location of the control point is updated as in Equation 10, below. The vector n is a unit normal to the plane of the toolpath. The distance for the update, u*, is obtained through a one-dimensional search based on a quadratic approximation to the cost as a function of u. Because simulation is required to evaluate the engagement metric, it is computationally advantageous to perform only local simulation when updating the control point location.

$$\mathbf{P}_i^* = \mathbf{P}_i + u^* \cdot (\mathbf{T}_i \times \mathbf{n}) \tag{10}$$

Additional Constraint to Maintain Geometric Feasibility

It is possible for a control point updating step based solely on the metrics to result in a geometrically infeasible toolpath. Specifically, a void may result in the resulting swept volume of the toolpath trajectory. Due to the wide breadth of potential solutions and the complex geometric requirements, it is relatively difficult to identify all possible problems a priori. However, one relatively simple constraint can be added to the updating step that maintains feasibility in the case of continuous volume regions. Figure 5 defines a convention for entry (α) and exit (γ) angles. In the figure, the direction of tool motion is vertical. Angles are measure positive in a counter-clockwise orientation. For the case of a continuous endmilling operation, the exit angle should remain close to 90 deg. An exception to this will occur towards the end of the cutting operation, when the remaining material becomes small (as in the above "tongue" configuration. One way to assure that no volume inadvertently remains is to place an additional constraint on the exit angle. Generally, it is sufficient to enforce the condition that the exit angle remain negative when climb milling (positive for conventional milling). Note that this constraint cannot be readily applied to complex regions where cutting must initiate and conclude multiple times.

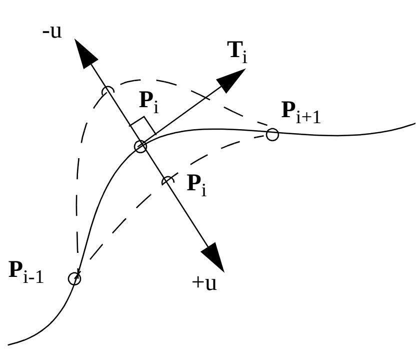


Figure 4. Search Direction for Control Point Updating

RESULTS

Two case studies demonstrating the above approach are documented in Figures 6 and 7. For each case, a standard contour-parallel toolpath was used as the starting point for the metric-based iterative improvement algorithm. The instantaneous engagement metric and the curvature metric were equally weighted in the application of the algorithm, and the algorithm was run for both spiral-in and spiral-out configurations. The tool-path trajectories in the figures represent the center line motion of the tool. To satisfy the geometric requirements, the peripheral (boundary) loop of the trajectory is not permitted to move. In other words, the optimized and initial tool-paths must machine identical volumes.

Plots of the instantaneous curvature and engagement before and after optimization are presented in the lower portions of Figures 6 and 7. Of immediate note is the reduction in the variation of the engagement angle after optimization. While significant variations remain present, the number and magnitude of these fluctuations has been significantly reduced. As expected, the sharp peaks in curvature have been greatly reduced in the improved toolpath. Also of significant interest is the difference between the spiral-in and spiral-out trajectories. From these figures, it is possible to consider explicitly the inherent trade-off between the two approaches. In the spiral-in approach, a large slotting pass is necessary around the periphery. This appears on the initial portion of the engagement plot. After the culmination of this slotting pass, the engagement variations are relatively small. In the spiral-out trajectory, the initial slotting is nearly nonexistent, as the path initiates with a spiraling pattern. However, it is not possible to completely eliminate spikes in the engagement due to

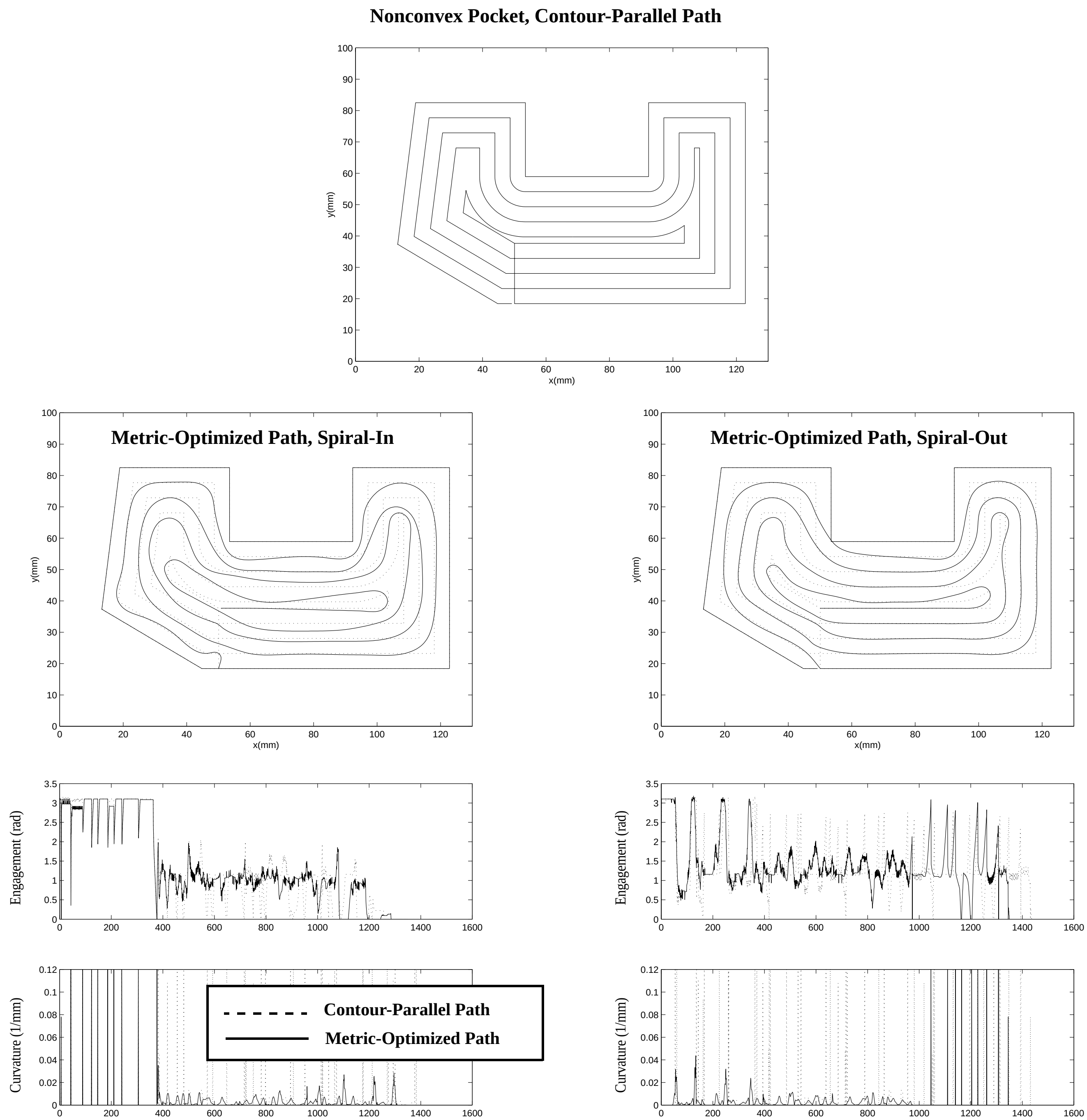


Figure 6. Tool-path Optimization Results, Nonconvex Pocket

the geometric constraints. As a result, numerous cornering spikes appear in the final pass around the periphery.

CONCLUSIONS

While it may not be possible to claim optimality of the tool-path trajectories resulting from the iterative improvement algorithm, it is evident that significant gains can be realized through a metric-based approach. Variations in instantaneous cutter engagement are a significant concern from a process stability and efficiency perspective. It is apparent from the engagement plots that the magnitude and number of problematic instantaneous engagement variations may be reduced. This is expected to result in an appreciable benefit to high-speed material removal operations.

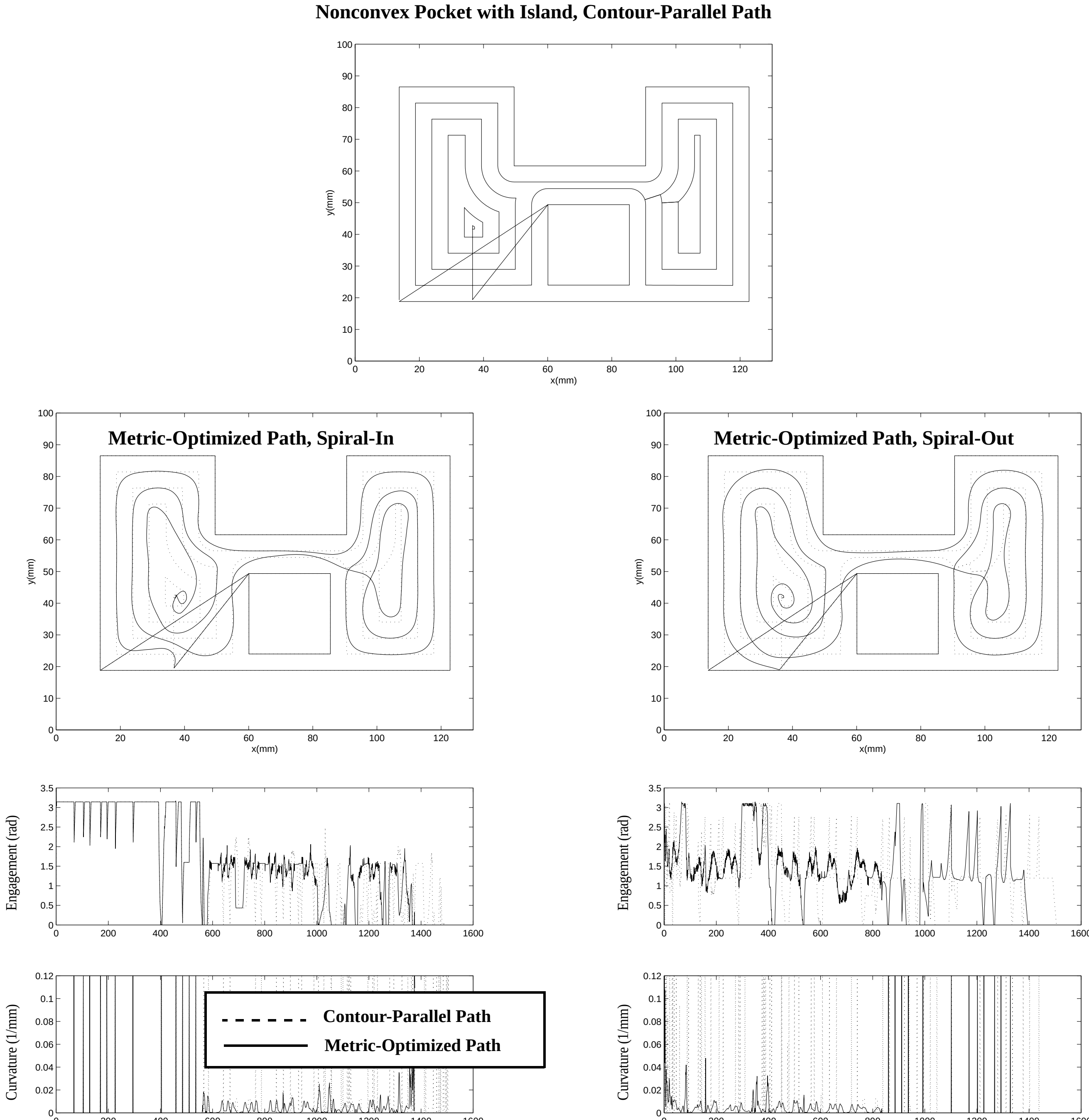


Figure 7. Tool-path Optimization Results, Nonconvex Pocket with Island

ACKNOWLEDGEMENTS

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REFERENCES

[Allock, 1994] Allock, A., “Fast-Moving Metal Removal Sets the Pace,” Machinery and Production Engineering, 152, 1994, p.43-48.

[Altintas and Budak, 1995]Altintas, Y., and Budak, E., “Analytical Prediction of Stability Lobes in Milling,” Annals of the CIRP, Vol. 45/ 1/1995 pp. 357-362.

[Chou and Yang, 1991] Chou, J-J., Yang, D. C. H., "Command Generation for Three-Axis CNC Machining," Journal of Engineering for Industry, Vol. 113, Aug. 1991, pp. 305-310.

[Davies et. al., 1998]Davies, M.A., Dutterer, B., Pratt, J.R., and Schaut, A.J., "On the Dynamics of High-Speed Milling with Long-Slender Endmills," Annals of the CIRP, Vol. 47/1/1998, pp. 55-60.

[Dews and Aspinwall, 1997] Dews, R. C., Aspinwall, D. K., "A Review of Ultra High Speed Milling of Hardened Steels," Journal of Materials Processing Technology 69, 1997, 1-17.

[Fallböhmer et al, 1996] Fallböhmer, P., Altan, T., Tönshoff, H., Nakagawa, T., "Survey of the Die and Mold Manufacturing Industry," Journal of Materials Processing Technology 59, 1996, 158-168.

[Farouki, 2000] Farouki, R. T., Tsai, Yi-Feng and Wilson, C. S., "Physical Constraints on Feedrates and Feed Accelerations along Curved Tool Paths," Computer Aided Geometric Design, 17, 2000, p337-359.

[Fussell et. al., 2001] Fussell, B K. Jerard, R B, and Hemmett, J.G., "Robust Feedrate Selection for 3-Axis NC Machining Using Discrete Models," ASME Journal of Manufacturing Science and Engineering, 123:2, 214-224, May 2001.

[Gough, 1990] Gough, J., "High Speed Machining for Toolmaking Applications," Precision Toolmakers 8(3), 1990, p.154.

[Hogan, 1999] Hogan, B., "No Speed Limits," Manufacturing Engineering 3, 1999, 66-78.

[Iwabe et al, 1989] Iwabe, H., Fujii, Y., Saito, K., and Kisinami, T., "Study of Corner Cut by End Mill - Analysis of Cutting Mechanism and New Cutting Method ant Inside Corner," J. of JSPE, Vol. 55, No. 5, 1989, pp841-846 (in Japanese).

[Jayaram et.al. 2000] Jayaram, S., S. G. Kapoor, and R. E. DeVor, "Analytical Stability Analysis of Variable Spindle Speed Machining," to appear in the Journal of Manufacturing Science and Engineering, Transactions of the ASME, 2000.

[Li et al, 1999] Li, D., Yang, J-B, and Biswal, M.P., "Quantitative Parametric Connections Between Methods for Generating Noninferior Solutions in Multiobjective Optimization," European Journal of Operational Research, Vol.117, 1999, pp. 84-89.

[Renton and Elbestawi, 2000] Renton, D., Elbestawi, M. A., "High Speed Servo Control of Multi-axis Machine Tools," International Journal of Machine Tools & Manufacture, Vol. 40, 2000, 539-559.

[Smith et al, 1991] Smith, S., Cheng, E., and Zamudia, C., "Computer-Aided Generation of Optimum Chatter-Free Pockets," Journal of Materials Processing Technology, Vol. 28, 1991, pp. 275-283.

[Smith and Tlusty, 1992] Smith, S., and Tlusty, J., "Stabilizing Chatter by Automatic Spindle Speed Regulation," Annals of CIRP, Vol 41/1/1992.

[Stori et. al., 1999] Stori, J. A., C. King, and P. K. Wright, "Integration of Process Simulation in Machining Parameter Optimization," ASME Journal of Manufacturing Science and Engineering, 121:1, 134-143, Feb. 1999.

[Stori and Wright, 2000] Stori, J. A. and P. K. Wright, "Constant Engagement Tool-Path Generation for Convex Geometries," Journal of Manufacturing Systems, 19:3, 172-184, 2000

[Stori and Wright, 2001] Stori, J. A. and Wright, P. K., "Parameter Space Decomposition for Selection of the Axial and Radial Depth of Cut in Endmilling," Journal of Manufacturing Science and Engineering, 123:4, 654-664, 2001.

[Stori and Ferreira, 2002] Stori, J. A., and Ferreria, P.M. "Design of a High-Speed Parallel Kinematics X-Y Table and Optimal Velocity Scheduling for High-Speed Machining," conditionally accepted to appear in *Transactions of NAMRI*, XXX, 2002.

[Tlusty et al, 1996] Tlusty, J., Smith, S., and Winfough, W.R., "Techniques for the Use of Long Slender End Mills in High-Speed Milling," Annals of the CIRP, Vol.45(1), 1996, 539-559.

[Tlusty et al, 1990] Tlusty, J., Smith, S., and Zamudio, C., and Zamudio, C., "New NC Routines for Quality in Milling," Annals of the CIRP, Vol. 39, No. 1, 1990, pp. 517-521.

[Tsai et al, 1991] Tsai, M.D., Takata, S., Inui, M., Kimura, F. and Sata, T., "Operation Planning Based on Cutting Process Models," Annals of the CIRP, Vol. 40, No. 1, 1991, pp.95-98.

[Wang, 1988] Wang, K.K., "Solid Modeling for Optimizing Metal Removal of Three-Dimensional NC End Milling," *J. Manuf. Syst*., 7:1, 57-65, 1988.

[Wang and Yang, 1993] Wang, F-C and Yang, D.C.H, "Nearly Arc-Length Parametrized Quintic Spline Interpolation for Precision Machining", Computer-Aided Design, Vol. 25, No.5, 1993 pp. 281-288.

[Winfough and Smith, 1995] Winfough, W.R., and Smith, S., "Automatic Selection of the Optimum Metal Removal Conditions for High-Speed Milling," Transactions of NAMRI, Vol. XXIII, pp. 163-168.

[Yan et al, 2000] Yan, X., Shirase, K., Hirao, M., and Yasui, T., "Evaluation and Improvement of Productivity in High-Speed NC Machining," ASME J. Manufacturing Science and Engineering, Vol. 122, No.3, 2000, pp.556-561.

[Yang and Sim, 1993] Yang, M.Y., and Sim, C.G., "Reduction of Machining Errors by Adjustment of the Ball-End Milling Process," Int. J. Prod. Res. 31:3, 665-689, 1993.

[Zelinski, 1999] Zelinski, P., "Boeing's One Part Harmony," MMS Online, August 1999, http://www.mmsonline.com/articles/089903.html.

[Zhao and Balachandran, 2001]Zhao, M.X., and Balachandran, B., "Dynamics and Stability of Milling Process," *Int. J. of Solids and Structures*, Vol 38 (2001) 2233-2248.